On the Substitutability of Local Newspaper, Radio, and Television Advertising in Local Business Sales

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Executive Summary

A current policy issue is whether there is a single local media market or several distinct local markets for newspaper, radio, and television advertising. An important step in defining a market is the estimation of elasticities of demand. In this study we present a model of local business behavior in purchasing advertising for use in sales activities. Using reported data on local radio and television advertising revenue contained in the 2001 BIA's *Master Access Database* and using data from the Newspaper Association of America on retail ad expenditures, estimates of elasticities of substitution, ordinary own- and cross-price elasticities are derived for a representative local business establishment. The estimated elasticities of substitution and the estimated ordinary cross-price elasticities suggest weak substitutability between local media. In addition, the ordinary demand elasticity for a retail ad in a newspaper is approximately unity and negative. Demands for both local radio and local television ads are inelastic. Finally, the data support the specified model of local business advertising. An important caveat to these results is that varying degrees of measurement error are associated with local radio revenue and revenue from retail ads in newspapers. Due to these limitations inherent in the underlying data, the results of this study cannot be considered conclusive. The results, however, are consistent with economic theory.

Introduction

Section 202 (h) of the Telecommunications Act of 1996 directs the Federal Communications Commission to review its broadcast ownership rules every two years in light of competitive developments. In conducting such reviews, the Commission may need to consider the extent to which established local media compete with each other for advertising dollars. The specific policy concern is whether there is a single local media market or several distinct local markets for newspaper, radio, and television adverting. This study is intended to contribute to that evaluation.

An important step in defining a market is the estimation of elasticities of demand. Therefore, in this study, we estimate elasticities of substitution for local newspaper, local radio, and local television advertising media. Ordinary own- and cross-price elasticities of demand are also estimated.

We note that this study evaluates the extent of substitutability of the three largest local media – television, radio, and daily newspapers. We do not attempt to address whether cable television or other advertising vehicles such as direct mail and outdoor advertising compete with local television, radio and daily newspapers for local advertising dollars.

Literature

The literature on inter-media substitutability in advertising can be divided into two broad categories. First, there is national advertising that is associated with national sales of product/services. Second, there is local media advertising by local businesses. Several authors have studied national advertising. McCullough and Waldon (1998)

examine the substitutability of network advertising and national spot advertising. They

assume an aggregate sales function and rely on duality in order to specify the share

equations of a translog cost function. Three share equations are considered: (1) the share

of network advertising, (2) the share of national spot television advertising, and (3) and

the share of a composite of all other advertising forms. Using McCann-Erickson which

spans the years 1960 to 1994, the authors estimate Morishima elasticities of substitution.

McCullough and Waldon find that television network advertising and national spot

television advertising are substitutes.

Silk, Klein, and Berndt (2001) use a translog model in their evaluation of inter-

media substitutability. In addition, their analysis permits for the complementary use of

media in national sales. Their translog model represents the advertising costs of national

sales. The share equations of the model are: (1) direct mail, magazines, newspapers, spot

radio, network television, network radio, spot television, and outdoor. The parameters of

the model are estimated based on national data for the period covering 1960 to 1994.

Estimates of own- and cross-price elasticities for the various media are presented. Silk,

Klein, and Berndt conclude that demands for the various media are inelastic. In addition,

they suggest that weak cross-media effects are consistent with industry practices in which

media planning is conducted as a multistage decision process. In this process inter-media

choices are made primarily on strategic and creative grounds. That is initial media buys

are likely to be a mix of media. Price-sensitivity is shown in subsequent intra-media

comparisons.

Sheldon, Jewell, and O'Brien (2000) investigate media substitutability and

economies of scale in advertising. These authors use a translog cost function to model

advertising cost of firms that manufacture beer. The inputs to the sale of beer are print, television, and radio. Both the translog function and media share equations are simultaneously estimated. Their data are on U.S. beer firms. The data are quarterly and cover the period 1983 to 1993. Morishima elasticities of substitution and price elasticities of demand are estimated. Sheldon, Jewell, and O'Brien find that all advertising media are substitutes in the selling of beer. In addition, evidence of diseconomies of scale in advertising is presented.

There is some research on inter-media substitutability in local advertising markets. Ekelund (1999) et al. evaluate whether a market for radio services by advertisers exists independently of other local advertising markets. Their method for identifying a radio market is based on the *DOJ Merger Guidelines.* Their approach consists of determining whether conditions exit such that a hypothetical monopolist could profitably and optimally raise price. To this end, Ekelund compares a calculated Lerner Index for radio with the reciprocal of the estimated own price elasticity of demand for radio. The issue of media substitutability arises in their specification of the functional from of demand. The authors estimated a double-log model with the log of revenue as a dependent variable. Bona fide explanatory variables are the prices of radio, television, newspaper, and total retail sales.

The unit of observation in this cross sectional study is an Arbitron market from BIA's *Master Access Database*. This BIA database also provided information on total radio revenues or expenditures by firms on radio in an Arbitron Market. All data are from the year 1995. The price of radio and television are taken from the SQAD database. The prices for both radio and television are both cost per rating point ("CPP"). Newspaper

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advertising prices are based on the price of a one-inch, black-and-white, one day advertisement in [Arbitron] markets. These price data are from the *Newspaper Advertising Source.*

Ekelund et al present estimates of own-price and cross-price elasticities. Comparing measures of operating margins, e.g., cash flow, and estimated own-prices elasticity of-2.101, these authors concluded that the radio market constitutes an antitrust market. On the issue of inter-media substitutability, demand cross-price elasticities are found to be significant at the ten percent level.

In a separate paper, Ekelund and the same co-authors conduct an analogous analysis of television. Again, the 1995 BIA *Master Access Database* is used to identify revenues from Designated Market Areas ("DMA"). Both television and radio ad price is the cost per rating point from SQAD. A double-log specification is used, where the log of television advertising revenue or expenditure is the dependent variable. Explanatory variables are the log of retail sales, the price of a television ad, the price of a radio ad, and the price of a one-inch, black-and-white, one day advertisement in DMA. Own- and cross- price elasticities are estimated. Both the cross-price elasticities of demand for radio and newspaper are positive, and the radio cross-price elasticity is positive at a five percent level of significance. However, in this analysis their calculated Lerner Index and the reciprocal of their estimated elasticity does not suggest that a hypothetical monopolist could profitably and optimally increase advertising prices.

The work of Ekelund et al. is limited in its generality for several reasons. Due to data limitations Ekelund could not differentiate between national radio (television) buys and local radio (television) buys within an Arbitron (DMA) market. As Silk et al. (2001)

argue, aggregating national and local market demand for media may hide different patterns and levels of inter-media substitutability/complementarity among more finely disaggregated components. In addition, our examination of the Newspaper Advertising Source suggests that the explanation of newspaper ad price is incomplete. The authors do not provide the details on which newspapers are included in the geographic area of analysis. Also details on the aggregation of newspaper prices into a single newspaper price are not explained. The Newspaper Advertising Source data suggest that numerous newspapers could have been included in the construction of the composite price for local newspapers. Alternatively, a single newspaper's price could have been used.

Theoretical Framework

Our focus is on the use of media by local businesses. Following Silk et al. (2001) we hypothesize that inter-media buys by local firms are made on strategic grounds and that a mix of media is used in advertising to local consumers.

We began by developing the derived demand for broadcast television, radio, and newspaper ads in a local market. Suppose that representative firm A provides product/service z in a local market. For simplicity of analysis, we assume that the geographic market for product z or area of geographic distribution of z is the same as the geographic market for television, radio, and newspaper ads. That is the geographic market for z, the local television ad market, the local radio ad market, and the local newspaper ad market all coincide. In these local media markets, firm A is a price taker, and, given media ad prices and the firm's media budget, firm A employs these local media to sell z.

We assume that all goods/services markets are competitive and that firms are Bertrand competitors. Entrepreneurs of firms, e.g., firm A, regard sales, however, as random variables. Sales of firms are random because consumers face uncertainty. Given perfect information on prices and no transportation costs, consumers must also decide where to spend. Consumers' choices of firms to patronize are made in accordance with consumer convenience in daily routine and with the degree of uncertainty in daily events.

We assume that all firms advertise under the belief that advertising reduces variability in sales and increases expected sales by reminding consumers of the existence of the firm. In this model newspaper, radio, and television are complementarily substitutable. Firm A uses a mix of media in its outreach to the targeted demographic group of potential buyers. If firm A buys television, it also buys some radio and some newspaper. As relative media prices change or as the effectiveness of a particular medium becomes know, firm A substitutes to a particular medium without necessarily abandoning the remaining media.

Following Veblen (1919), capital and labor employed in advertising (marketing) and in manufacturing are embedded in the constant return to scale production function of the firm. The unit of output, from the production of good z, is a *salable* good with specific characteristics, including shape, color, material composition, and consumer outreach through advertising (marketing) per unit for the period. Since there is constant returns to scale in the production of good z, there is a constant unit (marginal) cost,

denoted c_i for firm A. With Bertrand competition the market price is $p_z = c_z$.[1] Profits of firm A are $\Pi_A = p_z \times SALES_A - SALES_A \times c_z$.

We assume that the entrepreneur of firm A maximizes expected profit and that the entrepreneur of firm A prefers greater income from employment of her capital and labor to less income. Thus, firm A maximizes $E(\Pi_A) \Rightarrow \max E(SALES)$.[2] Expected sales are $\Phi(X_N - \gamma_N)^{\beta_N}(X_R - \gamma_R)^{\beta_R}(X_T - \gamma_T)^{\beta_T}$, where $\Phi > 0$ and where X_i, $i = \{N : newspaper, R : radio, T : television\}$, is the quantity of medium i purchased by firm A. The parameter, γ_i, is the base quantity of advertising of medium i, and the parameter $\beta_i > 0$ is the share of expenditures on medium i. The problem of firm A is

$$\max_{X_N, X_R, X_N} \Phi(X_N - \gamma_N)^{\beta_N}(X_R - \gamma_R)^{\beta_R}(X_T - \gamma_T)^{\beta_T} \text{ subject to } \sum_i p_i X_i = B_A.$$

The advertising budget of firm A is B_A which is in total outlays and which is embedded in the constant per unit cost of a *salable* product. The price per unit of medium i is p_i. Optimizing expected sales is simply the problem of optimizing the Stone-Geary utility function. The solution to this problem is the well known Linear Expenditure System ("LES"). Expenditures on medium i are

$$P_i X_i = \sum_j a_{ij} P_j + \beta_i B_A, \text{ where } a_{ij} = (1 - \beta_i)\gamma_i, i = j, \text{ and } a_{ij} = -\beta_i \gamma_j, i \neq j\text{ [3]}.$$

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[1] We can also think of the firm setting price at the intersection of marginal cost and the minimum of longrun average total cost.
[2] Because of constant return to scale, realized sales increase the entrepreneur's income.
[3] Alan P. Powell, *Empirical Analytics of Demand* (Massachusetts: Lexington Books, 1974), 36-38.

Empirical Specification and Estimation Technique

Leser's Transformation of the Linear Expenditure System ("LES")[4] permits us to directly estimate elasticities of substitution. Leser's transformation of the LES is

$$y_{it} = \sum_{j \neq i} \sigma_{ij}^o \Psi_{ijt} + \beta_i \bar{v}_t, \text{ where}$$

$$y_{it} = v_{it} - p_{it} x_i^o$$

$$\Psi_{ijt} = w_i^o v_j^o (\frac{p_{jt}}{p_j^o} - \frac{p_{it}}{p_i^o})$$

$$\bar{v}_t = m_t - \sum_{j=1}^{4} x_j^o p_{jt}$$

An observation, representative firm in a local market, is denoted by t. The price of medium i is p_{it}, and expenditure on medium i is v_{it}. The sample mean of expenditures on medium i is v_i^o. The sample mean values of the price of medium i and total expenditures are p_i^o and $m^o = B_A^o$, respectively. Let x_i^o be the mean quantity of medium i, and $x_i^o = \frac{v_i^o}{p_i^o}$. The average budget share at this coordinate set is $w_i^o = (\frac{p_i^o x_i^o}{m^o})$. The parameter σ_{ij}^o is the elasticity of substitution between medium i and j. The parameter β_i is the marginal expenditure share. In this system, homogeneity, symmetry, and adding up restrictions are imposed. Three equations are simultaneously estimated for this derived demand system:

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[4] Ibid., 62-66.

Due to singularity of the covariance matrix, which is a result of the adding-up restriction, we estimate using the econometric technique of Dhrymes (1984), (1987) and (1994)[5]. In this procedure, different variables can appear in different equations, and it not necessary to drop an equation from the system. After estimation, own price elasticities are calculated at sample means.

Data

Revenue data are from the 2001 BIA Master Access Database. Revenues are reported for television by DMA. Radio revenues are listed by Arbitron Radio Markets which are contained in DMAs.   At the time of the work of Ekelund et al. the BIA data contained an unknown mix of national and local advertising revenue for broadcast media. The 2001 BIA database contains, however, local advertising revenues for both television and radio broadcasting[6].

We took a random sample of DMAs. The sample contains 45 DMAs, and the DMAs are listed in Appendix A. We assume that local radio revenues of Arbitron Markets within a DMA are total local radio revenues for the DMA. Spot Quotation and Data, Inc. ("SQAD") data are the source of both local radio and television prices. Both radio and television prices are in units of cost per point ("CPP")[7].

Newspaper data are gathered and constructed from several sources. First, the Newspaper Association of America reports categories of newspaper advertising expenditures/revenue for the year 2000. The categories are: (1) national, (2) retail, and (3)

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[5] The 1994 paper is the published version of the 1984 working paper.
[6] The share of local revenue is reported for each DMA. Local television revenue is the product of the local share and total revenue. Shares of local revenue out of total revenue are provided for each radio market. Local radio revenue is the product of the local share and total radio revenue within the radio market.

classified advertising expenditures[8]. Retail advertising expenditures of $21,409 million are allocated across the entire universe of DMAs. Thus, retail advertising expenditure for a DMA is the product of $21,409 million and the allocation factor for that DMA. The allocation factor for a DMA is the ratio of the population over 16 years of age in a DMA to the population of the United States that is over 16 years of age[9]. Second, newspaper retail prices are from the *Newspaper Advertising Source*. Following Ekelund et al. (1999), price is the charge for a one-inch, black-and-white, one-day advertisement i.e., a Standard Advertising Unit ("SAU"). We used Monday through Friday-Saturday prices. For every DMA of our sample, we manually collected these price data on all identifiable newspapers. Since the number of newspapers varies by DMA, our measure of local retail price for a DMA is the mean price for newspapers in the DMA[10].

Our theory predicts the use of local advertising by a local business in the sale of its goods and/or services. This implies that we must translate local media revenues in a DMA into media expenditures of a representative local business in the DMA. For a given DMA, newspaper retail revenue is divided by business establishments in that DMA, and the result is local newspaper expenditures by a representative local business[11]. Expenditure on local radio advertising by a representative local business is the result of dividing local radio advertising revenue by business establishments in that DMA. The

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[7] SQAD reports CPP prices for both radio and television by quarter, and we use 4th quarter 2000 prices. Late news rates were used for television and evening rates were used for radio. The rates are for Adults 18+.

[8] Newspaper Association of America at website http://www.naa.org/artpage.cfm?AID=1566&SID=1002.

[9] The source of the population data is *Census 2000*, U.S. Census Bureau. Census 2000 population data are mapped into DMAs.

[10] Newspaper prices are presented by month. We used prices for mid-4th quarter (November) 2000.

[11] Business establishments in a county are from *County Business Patterns*, U.S. Census Bureau (1997). We mapped county business establishments into DMAs.

expenditure of a representative local business on local television advertising is the local television revenue in the DMA divided by the business establishments in the DMA.

Two dummy variables are created to control for competitive effects that are associated with the size or rank of a DMA. A dummy variable is assigned the value of one when a DMA's rank is less than or equal to 10 (top 10 DMAs), and, the variable is zero, otherwise. A second dummy variable is assigned the value of one when the rank of the DMA is between 11 and 50, and, the variable is zero, otherwise.

Results

Economic theory predicts that elasticities of substitution can vary from zero to infinity. For our function of expected sales, an elasticity of zero would indicate that there is no substitutability between two media since an elasticity of substitution[12] is a measure of pair-wise substitutability. Alternatively, an elasticity of substitution of infinity indicates that two media are perfect substitutes, e.g., television is a perfect substitute for radio.

Our results suggest weak substitutability between local media in the sales activities of local businesses. Specifically, with respect to the three media pairs we studied, we find: First, the elasticity of substitution between newspaper retail ads and local radio ads is 1.16936. [13] Compared to the theoretical limit of infinity for perfect substitutability this number is very small but statistically significant. Second, the elasticity of substitution between newspaper retail ads and local television ads is 0.91459.

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[12] Elasticities of substitution are technical parameters that reveal the shape or curvature of the expected sales function.

[13] We assumed symmetry, and therefore, the elasticity of substitution between local radio and newspapers retail ads is also 1.16936.

This number is also relatively small but statistically significant. <u>Third</u>, the elasticity of substitution between local radio and local television is 0.3094, and this elasticity is not statistically different from zero. These results are presented in Table 1 of Chart A.

Ordinary own- and cross-price elasticities are calculated at sample means. The results are presented in Table 2 of Chart A. Ordinary own-price elasticities of demand are negative. The ordinary own-price elasticity for a retail ad in a newspaper is -1.0406. Demands for both local radio advertising and local television advertising are inelastic. Local radio's own-price elasticity is -0.8245, and the own-price elasticity for television is -0.7960. Moreover, ordinary-cross price elasticities of demand also suggest weak substitutability and suggest that several media are gross complements[14]. The cross price elasticity between newspaper retail ads and local radio ads is 0.0178, while the cross-price elasticity between local radio and newspaper retail ads is 0.0981. The cross-price elasticities for newspaper retail ads and local television ads are negative. In addition, the cross-price elasticities for television and radio are negative. All ordinary cross-price elasticities are presented in Table 2 of Chart A.

There are certain limitations associated with this study as a result of the underlying data that informs our study. Specifically, the study contains varying degrees of measurement errors on certain variables. First, local radio revenue within a DMA is incomplete because BIA does not report revenue data for all radio stations but only for Arbitron markets, which generally are the larger markets. Second, newspaper retail expenditure in a DMA results from an allocation process that assigns retail revenues to DMAs based on a DMA's share of population 16 years or older. This methodology is

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[14] For the Linear Expenditure System all goods should a priori be gross complements. The data support the LES specification.

used because there does not appear to be a source for newspaper retail revenues by DMA. In addition, the mapping of Census demographic data into DMAs involves some judgment because some counties are contained in multiple DMAs. Moreover, the reach of some newspapers may extend beyond the assigned DMA, and business establishments outside the DMA could have spent advertising resources on newspaper ads which are assigned to the DMA. Nonetheless, acknowledging the limitations of the data, the results do not appear unreasonable or inconsistent with economic theory.

Summary and Conclusion

This study examines the substitutability of local newspaper, radio, and television advertising in the sales activities of local businesses. A local business within a DMA is hypothesized to maximize expected sales by selecting an optimal mix of newspaper retail ads, local radio ads, and local television ads. The expected sales function for goods and/or services of the local business is a scalar multiple of the Stone-Geary utility function, and the resulting derived demand for local media is the Linear Expenditure System. Leser's transformation of the LES permits the direct and linear estimation of elasticities of substitution. Since the LES gives rise to a singular covariance matrix, the econometric technique of Dhrymes (1984) and (1994) is used to estimating this singular system of equations. A random sample of 45 DMAs is used in the study. Local radio and local television data are taken from the BIA Master Access Database. Newspaper retail revenues for DMAs are constructed from data published by the Newspaper Association of America.

Estimates of elasticities of substitution, ordinary own- and cross-price elasticities are derived for a representative local business establishment. The estimated elasticities of substitution and the estimated ordinary cross-price elasticities suggest weak substitutability between local media. The elasticities of substitution are relatively small, and the elasticity of substitution between radio and television is not statistically different from zero. The ordinary cross-price elasticities for newspaper retail ads and local radio ads are small but positive. The ordinary cross-price elasticities for newspaper retail ads and local television ads are negative. In addition, the ordinary cross-price elasticities for local radio ads and local television ads are negative. This suggests that local newspaper and television ads are complementary inputs in the sales efforts of local businesses. These results also suggest that local radio and television ads are also complementary inputs.

The following caveat must be acknowledged when considering this study in communications policy. There are limitations inherent in the underlying data. For example, local radio ad expenditures are not total expenditures on radio within a DMA because total local radio revenue is not reported. In addition, local newspaper ad expenditures are constructed through an allocation process that introduces some degree of measurement error. The allocation process used in this paper assigns retail newspaper advertising expenditures in each DMA based on a DMA's share of population 16 years or older. The estimated elasticities are not, however, inconsistent with economic theory and do not appear unreasonable.

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	Chart A					
	Advertising in the Generation					
	of Local Business Sales					
			Table 1			
	Marginal	Elasticity	Elasticity			
	Expenditure	of	of	Dummy	Dummy	
	Share	Substitution	Substitution	TOP	DMA	R-SQ
Local		Radio	Television	10 DMAs	11-50	
Newspaper : Retail Ad	0.48494	1.16936	0.91459	44.69234	-286.4512	0.9683
	(0.01698)	(0.0984)	(0.0644)	(417.597)	(156.519)	
Radio Advertising	0.19168		0.3094	282.5515	103.5607	0.9744
	(0.011508)		(.3087)	(266.2748)	(98.5364)	
Television Advertising	0.32337			-327.2439	182.8851	
	(0.01246)			(300.0201)	(109.2786)	0.9864

Table 2

Local Advertising: Ordinary Own- and Cross-Price Elasticities

	Newspaper	Radio	Television
Newspaper	-1.04062	0.01780	-0.05823
Radio	0.09805	-0.82450	-0.22436
Television	-0.00440	-0.12399	-0.79601

*Standard errors in parentheses

	Appendix A: Random sample of DMAs		
	DMA		DMA
OBS	MARKET	OBS	MARKET
1	Baton Rouge, LA	26	Meridian, MS
2	Beaumont-Port Arthur, TX	27	Miami - Ft. Lauderdale, FL
3	Billings, MT	28	Nashville, TN
4	Boise, ID	29	New York, NY
5	Boston, MA	30	Oklahoma City, OK
6	Burlington, VT-Plattsburgh, NY	31	Omaha, NE
7	Casper-Riverton, WY	32	Phoenix, AZ
8	Chattanooga, TN	33	Pittsburgh, PA
9	Des Moines-Ames, IA	34	Portland-Auburn, ME
10	Detroit, MI	35	Rapid City, SD
11	Duluth, MN-Superior, WI	36	Reno, NV
12	El Paso, TX	37	Rochester, NY
13	Ft. Myers-Naples, FL	38	Salt Lake City, UT
14	Harlingen-Weslaco-McAllen-Brownsville, TX	39	San Diego, CA
15	Houston, TX	40	Springfield-Holyoke, MA
16	Indianapolis, IN	41	Syracuse, NY
17	Kansas City, KS-MO	42	Terre Haute, IN
18	Lafayette, IN	43	West Palm Beach-Ft. Pierce, FL
19	Lansing, MI	44	Wheeling, WV- Steubenville, OH
20	Las Vegas, NV	45	Wichita - Hutchinson, KS
21	Lexington, KY		
22	Little Rock-Pine Bluff, AR		
23	Louisville, KY		
24	Madison, WI		
25	Medford-Klamath Falls, OR		

Appendix 1: A Note on Estimating Leser's Transformation of the LES

In the procedure of Dhrymes (1987) and (1994), there is a condition requiring each variable of the system to appear in at least two equations. Except for the variable $\bar{v}_t = m_t - \sum_{j=1}^{n} x_j^o p_{jt}$, no equation of Leser's Transformation of the LES contains a set of variables common to any other equation. Bush (1994) provided specific conditions where Leser's Transformation of the LES can be estimated using the generalized inverse estimation procedure. Through the requirements that partial elasticities of substitution be symmetrical, i.e., $\sigma_{ij}^o = \sigma_{ji}^o$ and that the constraints imposed on the parameters of the system satisfy the adding-up condition, each variable of the transformed system appears in at least two equations of that system. This is demonstrated through Lemma 1 and Proposition 1.

Lemma 1. Let $\Psi_{ijt} = w_i^o v_j^o (\frac{p_{jt}}{p_j^o} - \frac{p_{it}}{p_i^o})$, where $w_i^o = (\frac{p_i^o x_i^o}{m^o})$ and $v_j^o = p_j^o x_j^o$.

Then $\Psi_{ijt} = -\Psi_{jit}, i.e., \Psi_{ij} = -\Psi_{ji}$, where $\Psi_{ij} = [\Psi_{ijt}, K, \Psi_{ijT}]'$.

Proof.

$$\Psi_{ijt} = \frac{p_i^o x_i^o}{m^o} p_j^o x_j^o (\frac{p_{jt}}{p_j^o} - \frac{p_{it}}{p_i^o}) = -\frac{p_j^o x_j^o}{m^o} p_i^o x_i^o (\frac{p_{it}}{p_i^o} - \frac{p_{jt}}{p_j^o}).$$

$$\Psi_{ijt} = -w_j^o v_i^o (\frac{p_{it}}{p_i^o} - \frac{p_{jt}}{p_j^o}) = -\Psi_{jit}.$$

$$\Psi_{ijt} = -\Psi_{jit} \Rightarrow \Psi_{ij} = -\Psi_{ji}. \qquad \qquad \text{q.e.d.}$$

Proposition 1. Suppose that we have Leser's Transformation of the LES.

$$y_{it} = \sum_{j \neq i} \sigma_{ij}^o \Psi_{ijt} + \beta_i \bar{v}_t \qquad (1)$$

$$\text{where } y_{it} = v_{it} - p_{it} x_i^o$$

$$\Psi_{ijt} = w_i^o v_j^o \left(\frac{p_{jt}}{p_j^o} - \frac{p_{it}}{p_i^o} \right)$$

$$\bar{v}_t = m_t - \sum_{j=1}^n x_j^o p_{jt} .$$

If

i) Ψ_{ij} replaces Ψ_{ji} in the j th equation and α is the parameter associated with Ψ_{ij} in the j th equation; *ii)* the adding-up constraints are imposed on the parameters of the system; and *iii)* $\sigma_{ij}^o = \sigma_{ji}^o$ (elasticities of substitution are symmetrical), then system (1) is preserved.

Proof.

Replacing Ψ_{ji} by Ψ_{ij} in equation j and recognizing that Ψ_{ij} now occurs <u>only</u> in the i th and j th equations,

$$\mathbb{N}$$

$$y_i = \Lambda + \sigma_{ij}^o \Psi_{ij} + \Lambda + \sigma_{in}^o \Psi_{in} + \beta_i \bar{V}$$

$$y_j = \Lambda + \alpha \Psi_{ij} + \Lambda + \sigma_{jn}^o \Psi_{jn} + \beta_j \bar{V}$$

$$\mathbb{N}$$

The adding-up constraints $\Rightarrow \alpha = -\sigma_{ij}^o$ and $\sum_{j=1}^n \beta_j = 1.$ Thus,

$$\mathbb{N}$$

$$y_i = \Lambda + \sigma_{ij}^o \Psi_{ij} + \Lambda + \sigma_{in}^o \Psi_{in} + \beta_i \bar{V}$$

$$y_j = \Lambda - \sigma_{ij}^o \Psi_{ij} + \Lambda + \sigma_{jn}^o \Psi_{jn} + \beta_j \bar{V}$$

$$\mathbb{N}$$

$$\Rightarrow$$

$$\mathbb{N}$$

$$y_i = \Lambda + \sigma_{ij}^o \Psi_{ij} + \Lambda + \sigma_{in}^o \Psi_{in} + \beta_i \bar{V}$$

$$y_j = \Lambda + \sigma_{ij}^o \Psi_{ji} + \Lambda + \sigma_{jn}^o \Psi_{jn} + \beta_j \bar{V}$$

$$\mathbb{N}$$

by Lemma 1.

Since $\sigma_{ij}^o = \sigma_{ji}^o$,

$$N$$

$$y_i = \Lambda + \sigma_{ij}^o \Psi_{ij} + \Lambda + \sigma_{in}^o \Psi_{in} + \beta_i \bar{V}$$

$$y_j = \Lambda + \sigma_{ji}^o \Psi_{ji} + \Lambda + \sigma_{jn}^o \Psi_{jn} + \beta_j \bar{V}$$

$$N$$

which is system (1).   q.e.d